***EVD***

Eigenvalue decomposition or sometimes spectral decomposition is the factorization of a matrix into a canonical form, whereby the matrix is represented in terms of its eigenvalues and eigenvectors. Only diagonalizable matrices can be factorized in this way.

**A** (non-zero) vector v of dimension N is an eigenvector of a square (NxN) matrix A if and only if it satisfies the linear equation

**A\*v**=*lambda*\***v**

where *lambda* is a scalar, termed the eigenvalue corresponding to **v**.

Let **A** be a square (NxN) matrix with N linearly independent eigenvectors, q*i* (*i* = 1, …, N). Then **A** can be factorized as

where **E** is the square (NxN) matrix whose *i*th column is the eigenvector q*i* of **A** and **L** is the diagonal matrix whose diagonal elements are the corresponding eigenvalues, i.e., **D***ii* = *lambdai*.

**Useful facts regarding eigenvalues**

1. The product of the eigenvalues is equal to the determinant of **A**
2. The sum of the eigenvalues is equal to the trace of **A**

***SVD***

Formally, the singular value decomposition of an mxn real or complex matrix **M** is a factorization of the form

where **U** is an mxm real or complex unitary matrix, **S** is an mxn diagonal matrix with nonnegative real numbers on the diagonal, and **V**\* (the conjugate transpose of V) is an nxn real or complex unitary matrix. The diagonal entries **S***ii* are known as the ***singular values*** of **M**. The m columns of **U** and the n columns of **V** are called the ***left singular vectors*** and ***right singular vectors*** of **M**, respectively.

*Singular value decomposition* and *Eigenvalues decomposition* are closely related:

* The left singular vectors of **M** are eigenvectors of **MM**\*
* The right singular vectors of **M** are eigenvectors of **M**\***M**
* The non-zero singular values of **S** are the square roots of the non-zero eigenvalues of **M**\***M** or **MM**\*

Applications which employ the **SVD** include computing the pseudoinverse, least squares fitting of data, matrix approximation, and determining the rank, range and null space of a matrix.